

values than were any of the other atomic parameters.

The present structure and that of von Stackelberg and Paulus are related geometrically to the structure of fluorite; this is illustrated in Fig. 2. Consider one fluorite-type unit cell with  $\text{As}^{3-}$  replacing  $\text{Ca}^{2+}$  and  $\text{Cd}^{2+}$  replacing  $\text{F}^-$ , but with two cube-diagonally opposite fluorite sites vacant. The unit cell of Goodyear and Steigmann contains 16 such units in the orientation shown in Fig. 2(a), the tail and head of the arrow being situated at the upper and lower vacant sites, respectively, in one unit. Fig. 2(b) shows the orientation of such units in the structure of Stackelberg and Paulus, there being 4 units in one unit cell of structure.

$\text{Cd}-\text{As}$  and  $\text{As}-\text{As}$  bond lengths in each type of As tetrahedron are given in Table 4. The variation in some of the  $\text{As}-\text{As}$  distances shows that there is appreciable distortion from cubic close-packing of As ions.

We wish to thank Dr D. W. G. Ballentyne and Mr G. L. Bucknell for providing us with specimens and Dr P. J. Wheatley, of Monsanto Research S.A., for permitting us to use some of his computer programs.

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## A Refinement of the Structure of Calcium Hexa-antipyrine Perchlorate and a Comparative Study of Some Metal Hexa-antipyrine Perchlorates

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Calcium hexa-antipyrine perchlorate,  $\text{Ca}(\text{C}_{11}\text{H}_{12}\text{ON}_2)_6(\text{ClO}_4)_2$ , is isomorphous with the corresponding magnesium and lead compounds and crystallizes in the trigonal space group  $P\bar{3}$  with one formula unit in an elementary cell of dimensions  $a=14.33$  and  $c=9.78 \text{ \AA}$ . The structure was refined by structure-factor least-squares method to an  $R$  value of 0.118 for 1131 observed reflexions. The structural features of magnesium, calcium and lead hexa-antipyrine perchlorates are compared. The coordination of antipyrine oxygen atoms around the metal ion is octahedral in all three cases; this octahedron becomes elongated along the  $\bar{3}$  axis as the size of the metal ion increases. The nature of the metal–oxygen bonds is discussed on the basis of the known electronegativity coefficients of the relevant atoms and the observed metal–oxygen distances. In the antipyrine molecule, both the phenyl and the pyrazolone rings are planar and are inclined with respect to each other by angles varying from 62 to 68°. An attempt has been made to explain the observed bond lengths in the pyrazolone ring in terms of the major canonical structures proposed.

### Introduction

The present study forms part of a program of systematic X-ray investigation of some metal antipyrine compounds undertaken in this laboratory to study the nature of the metal–oxygen bonding in these and also to elucidate the geometry of the antipyrine ring system.

Independent analyses of the isostructural lead and magnesium hexa-antipyrine perchlorates have already been reported (Vijayan & Viswamitra, 1966, 1967). Divalent calcium has an ionic radius ( $r=0.99 \text{ \AA}$ ) mid-way between those of  $\text{Mg}^{2+}$  ( $r=0.65 \text{ \AA}$ ) and  $\text{Pb}^{2+}$  ( $r=1.21 \text{ \AA}$ ) (Wells, 1962) and hence it was thought worth while to analyse the structure of the calcium compound to facilitate a study of the changes introduced in the molecular geometry and packing as a result of the change in the size of the central metal ion. In addition, it would

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enhance the knowledge about the molecular geometry of antipyrine already derived from the analysis of the magnesium compound.

## **Experimental**

Morphological and X-ray examination showed that calcium hexa-antipyrine perchlorate,



crystallizes in the trigonal space group  $P\bar{3}$  with one formula unit in an elementary cell of dimensions  $a=14.33$ ,  $c=9.78$  Å. The measured and calculated densities were 1.330 and 1.310 g.cm<sup>-3</sup> respectively (Vijayan & Viswamitra, 1965).

Intensity data were recorded with use of an approximately cylindrical specimen of mean radius 0.25 mm with Cu  $K\alpha$  radiation on multiple film equi-inclination Weissenberg photographs about the  $c$  axis for reciprocal levels  $hkl$ ,  $l=0$  through 7.  $h0h$  zonal data were also collected likewise with a crystal cut along the  $a_2$  axis and ground approximately to the same radius. The intensities were visually estimated by comparison with calibrated strips. Out of a total number of 2648 independent reflexions in the copper sphere 2181 were recorded of which 1131 were in the measurable range.

The data were corrected for Lorentz and polarization factors and the variation in spot shape in upper level Weissenberg photographs with the use of Nambudiri's program (Nambudiri, Kannan & Vijayan, 1966) on the Elliott 803-B computer at the Hindustan Aeronautics Ltd, Bangalore. Absorption corrections corresponding to  $\mu r = 0.5$  ( $\mu = 20.1 \text{ cm}^{-1}$  for  $\lambda = 1.5418 \text{ \AA}$ ;  $r = 0.25 \text{ mm}$ ) were also applied to the intensities (*International Tables for X-ray Crystallography*, 1959). The intensities were put on the same scale by comparison with the data from the isomorphous magnesium compound.

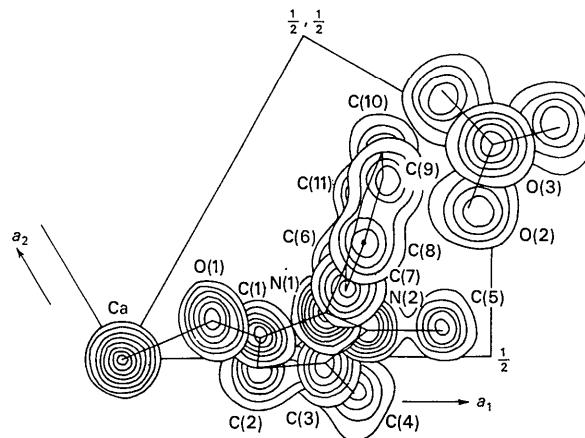
## Refinement

As the ionic radius of  $\text{Ca}^{2+}$  is mid-way between those of  $\text{Mg}^{2+}$  and  $\text{Pb}^{2+}$ , the average values of the positional coordinates and the equivalent isotropic temperature factors found in the magnesium and the lead compounds were used as the initial parameters in refinement procedures. Structure-factor calculations with these parameters gave a residual index of 0.230 for  $hki0$  and  $h0hl$  reflexions. The structure was refined with the use of zonal data to an  $R$  index of 0.139 by three cycles of isotropic structure-factor least-squares calculations.

The first three-dimensional SFLS cycle computed on the Elliott 803-B machine at Bangalore with the parameters refined from zonal data gave an  $R$  value of 0.169 for 1131 measured reflexions. The indicated shifts in the individual isotropic temperature factors were of the order of the corresponding standard deviations and hence individual anisotropic temperature factors of the form

$$\exp [-(b_{11}h^2 + b_{22}k^2 + b_{33}l^2 + b_{12}hk + b_{23}kl + b_{13}hl)]$$

were used in further refinement cycles carried out on the CDC-3600 computer system installed in the Tata Institute of Fundamental Research, Bombay. The first anisotropic structure-factor calculations yielded a discrepancy index of 0.144. In six diagonal SFLS cycles this index was reduced to 0.118. In the final cycle all the indicated shifts except one were less than the corresponding standard deviations, the average and the maximum shifts being  $0.29\sigma$  and  $1.10\sigma$  respectively.



**Fig. 1.** Final three-dimensional electron density projection along the *c* axis. Contours are at intervals 1, 2, 3, ... e. $\text{\AA}^{-3}$  for C, N and O and 1, 5, 10, ... e. $\text{\AA}^{-3}$  for Ca. The position of Cl is below O(3).

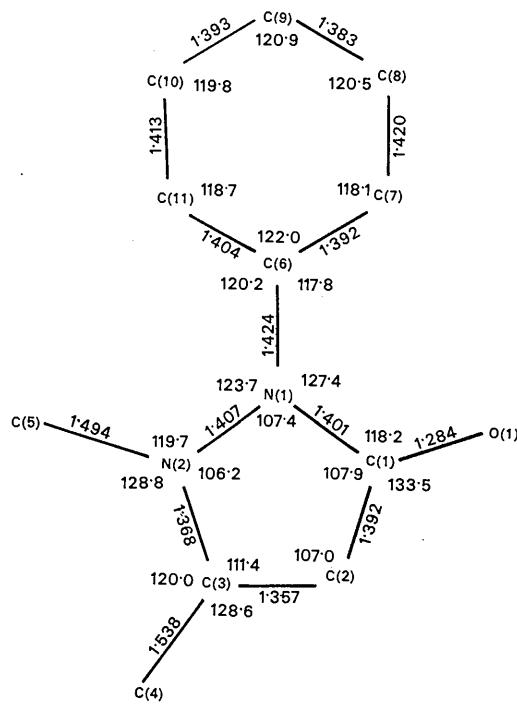


Fig. 2. Dimensions of the antipyrine molecule.

In the above calculations, reflexions with  $kF_o$  greater than 6 and less than 35 were given unit weights whereas those with  $kF_o$  less than 6 and greater than 35 were given half weights. Five low angle reflexions with  $kF_o$  greater than 50 were assigned zero weights as they were believed to be affected by extinction. The  $F_o$ 's were scaled to correspond to half the contents of the unit cell. Scattering factors for all the atoms were computed by means of the nine-term analytical expressions given by Cromer & Waber (1965).

The computer programs used in this analysis were those developed in this laboratory (Nambudiri *et al.*, 1966; Kannan, 1965; Vijayan, 1966).

### Results

In Table 1 are listed the final positional parameters and their standard deviations calculated from least-squares residues. The components of the anisotropic temperature factor tensors and the equivalent isotropic temperature factors (Hamilton, 1959) are given in Table 2. Table 3 lists the magnitudes and direction cosines with respect to  $a^*$ ,  $b^*$  and  $c^*$  axes of the principal axes of thermal vibration ellipsoids. The bond lengths and valency angles in the structure together with their standard deviations are listed in Table 4. The observed and calculated structure factors (unobservables excluded) are given in Table 5.

A final Fourier synthesis was computed and superposed sections of the corresponding electron density distribution are given in Fig. 1. Atomic positions are also indicated in the Figure. A difference Fourier synthesis was also computed in an attempt to locate hydrogen atoms, but their positions could not be determined unambiguously from the difference diagram.

### Description of the structure

#### Coordination around $\text{Ca}^{2+}$

The  $\text{Ca}^{2+}$  ion in the structure occupies a position of  $\bar{3}$  symmetry at the origin of the unit cell. The octa-

Table 1. Fractional positional parameters

Standard deviations are given in parentheses

|       | <i>x</i>     | <i>y</i>      | <i>z</i>       |
|-------|--------------|---------------|----------------|
| Ca    | 0            | 0             | 0              |
| Cl    | 0.66667 (0)  | 0.33333 (0)   | -0.51063 (50)  |
| O(1)  | 0.14833 (49) | 0.06146 (50)  | -0.14038 (69)  |
| O(2)  | 0.58767 (81) | 0.23044 (79)  | -0.56196 (99)  |
| O(3)  | 0.66667 (0)  | 0.33333 (0)   | -0.36506 (130) |
| N(1)  | 0.30982 (54) | 0.07315 (54)  | -0.19791 (74)  |
| N(2)  | 0.35251 (58) | 0.04850 (57)  | -0.31288 (80)  |
| C(1)  | 0.20066 (66) | 0.03567 (66)  | -0.22424 (96)  |
| C(2)  | 0.17568 (69) | -0.01611 (68) | -0.35049 (99)  |
| C(3)  | 0.26776 (73) | -0.00843 (71) | -0.39980 (95)  |
| C(4)  | 0.28439 (93) | -0.05587 (93) | -0.53256 (114) |
| C(5)  | 0.45477 (79) | 0.04538 (77)  | -0.29987 (114) |
| C(6)  | 0.37424 (66) | 0.14462 (65)  | -0.09207 (92)  |
| C(7)  | 0.35236 (81) | 0.10698 (78)  | 0.04197 (107)  |
| C(8)  | 0.41435 (89) | 0.17872 (90)  | 0.14835 (114)  |
| C(9)  | 0.49437 (82) | 0.28278 (83)  | 0.11851 (116)  |
| C(10) | 0.51461 (81) | 0.32012 (80)  | -0.01584 (122) |
| C(11) | 0.45479 (74) | 0.25015 (74)  | -0.12379 (106) |

hedral coordination of antipyrine oxygen atoms around  $\text{Ca}^{2+}$  is, thus, determined by the symmetry of the space group. The six  $\bar{3}$  equivalent  $\text{Ca}^{II}-\text{O}$  distances have a value of  $2.304 \pm 0.006 \text{ \AA}$ , which compares well with those found in other compounds (*International Tables for X-ray Crystallography*, 1962).

#### The antipyrine group

The dimensions of the antipyrine molecule in the structure are shown in Fig. 2.

The equations of the mean planes of the phenyl and the pyrazolone rings with respect to orthogonal axes  $X'=x+y \cos \gamma$ ,  $Y'=y \sin \gamma$  and  $Z'=z$  and the displacements of the ring atoms from and their standard deviations normal to the mean planes are given in Table 6. The r.m.s. displacements of the atoms in the phenyl ring and of those in the pyrazolone ring from their respective mean planes are  $0.005 \text{ \AA}$  and  $0.015 \text{ \AA}$  respectively. The mean plane of the phenyl ring makes an angle of  $62^\circ 31'$  with that of the pyrazolone ring.

Table 2. Anisotropic thermal parameters ( $\times 10^5$ ) and the equivalent isotropic temperature factors

|       | $b_{11}$ | $b_{22}$ | $b_{33}$ | $b_{12}$ | $b_{23}$ | $b_{13}$ | <i>B</i>             |
|-------|----------|----------|----------|----------|----------|----------|----------------------|
| Ca    | 311      | 311      | 498      | 311      | 0        | 0        | $1.91 \text{ \AA}^2$ |
| Cl    | 970      | 970      | 1062     | 970      | 0        | 0        | 5.34                 |
| O(1)  | 898      | 885      | 1408     | 873      | -188     | -88      | 5.48                 |
| O(2)  | 1881     | 1796     | 2347     | 1635     | -27      | 112      | 10.82                |
| O(3)  | 1851     | 1851     | 774      | 1851     | 0        | 0        | 8.59                 |
| N(1)  | 700      | 737      | 920      | 628      | -125     | 26       | 4.25                 |
| N(2)  | 784      | 775      | 1079     | 636      | -39      | -113     | 4.77                 |
| C(1)  | 674      | 585      | 1094     | 453      | -60      | 77       | 4.22                 |
| C(2)  | 749      | 657      | 1090     | 529      | -35      | 0        | 4.51                 |
| C(3)  | 723      | 873      | 828      | 687      | 52       | 228      | 4.49                 |
| C(4)  | 1449     | 1172     | 1223     | 1134     | -109     | 321      | 7.18                 |
| C(5)  | 843      | 876      | 1740     | 891      | -82      | 99       | 7.04                 |
| C(6)  | 630      | 576      | 870      | 474      | 45       | 210      | 3.76                 |
| C(7)  | 978      | 872      | 1058     | 815      | 125      | 285      | 5.30                 |
| C(8)  | 1054     | 1158     | 1189     | 975      | 72       | 188      | 6.25                 |
| C(9)  | 1071     | 952      | 1528     | 1023     | -64      | -158     | 6.09                 |
| C(10) | 922      | 873      | 1737     | 960      | 12       | -246     | 5.81                 |
| C(11) | 683      | 796      | 1474     | 745      | 51       | -161     | 4.88                 |

Table 3. Magnitudes and direction cosines of the principal axis of the vibration ellipsoids

| Atom  | Axis | $B_i$               | $g_{i1}$ | $g_{i2}$ | $g_{i3}$ |
|-------|------|---------------------|----------|----------|----------|
| Ca    | 1    | 1.92 Å <sup>2</sup> | 1        | 0.5000   | 0        |
|       | 2    | 1.92                | 0        | 0.8660   | 0        |
|       | 3    | 1.91                | 0        | 0        | 1        |
| Cl    | 1    | 5.98                | 1        | 0.5000   | 0        |
|       | 2    | 5.98                | 0        | 0.8660   | 0        |
|       | 3    | 4.06                | 0        | 0        | 1        |
| O(1)  | 1    | 5.88                | 0.2299   | 0.7251   | -0.6714  |
|       | 2    | 5.60                | -0.9028  | -0.0946  | 0.1235   |
|       | 3    | 4.96                | 0.3636   | 0.6822   | 0.7308   |
| O(2)  | 1    | 12.67               | 0.6415   | -0.3385  | 0.0953   |
|       | 2    | 10.86               | -0.7625  | -0.9410  | -0.0303  |
|       | 3    | 8.94                | 0.0847   | -0.0038  | -0.9950  |
| O(3)  | 1    | 11.41               | 1        | 0.5000   | 0        |
|       | 2    | 11.41               | 0        | 0.8660   | 0        |
|       | 3    | 2.96                | 0        | 0        | 1        |
| N(1)  | 1    | 5.10                | 0.3173   | -0.6365  | 0.2374   |
|       | 2    | 4.22                | -0.9483  | -0.7394  | 0.0832   |
|       | 3    | 3.42                | 0.0036   | 0.2196   | 0.9679   |
| N(2)  | 1    | 5.71                | 0.5370   | -0.4547  | -0.1191  |
|       | 2    | 4.59                | -0.7501  | -0.8404  | 0.3854   |
|       | 3    | 4.02                | 0.3859   | 0.2948   | 0.9150   |
| C(1)  | 1    | 5.15                | 0.6237   | -0.3035  | 0.3259   |
|       | 2    | 4.07                | -0.2673  | 0.0378   | 0.9431   |
|       | 3    | 3.45                | 0.7346   | 0.9521   | 0.0665   |
| C(2)  | 1    | 5.48                | 0.6702   | -0.3058  | 0.0575   |
|       | 2    | 4.18                | -0.2061  | -0.1992  | 0.9722   |
|       | 3    | 3.89                | 0.7130   | 0.9310   | 0.2269   |
| C(3)  | 1    | 5.87                | 0.1246   | -0.7935  | 0.0892   |
|       | 2    | 4.63                | -0.9309  | -0.5932  | -0.3343  |
|       | 3    | 2.96                | 0.3435   | 0.1359   | -0.9383  |
| C(4)  | 1    | 9.88                | 0.8113   | -0.0697  | 0.2014   |
|       | 2    | 7.23                | -0.5706  | -0.9941  | 0.0681   |
|       | 3    | 4.45                | 0.1274   | -0.0837  | -0.9772  |
| C(5)  | 1    | 6.77                | 0.1231   | -0.1314  | 0.9671   |
|       | 2    | 5.39                | -0.6962  | -0.9663  | -0.0759  |
|       | 3    | 4.95                | 0.7072   | -0.2214  | -0.2430  |
| C(6)  | 1    | 4.70                | 0.6603   | -0.2550  | 0.3278   |
|       | 2    | 3.58                | -0.4871  | -0.8649  | -0.4979  |
|       | 3    | 3.00                | 0.5717   | 0.4323   | -0.8029  |
| C(7)  | 1    | 6.64                | 0.7889   | -0.1021  | 0.2214   |
|       | 2    | 5.43                | -0.5277  | -0.9710  | -0.2343  |
|       | 3    | 3.82                | 0.3151   | 0.2164   | -0.9466  |
| C(8)  | 1    | 7.74                | 0.2371   | -0.7215  | 0.0526   |
|       | 2    | 6.52                | -0.9468  | -0.6837  | -0.2113  |
|       | 3    | 4.45                | 0.2178   | 0.1091   | -0.9760  |
| C(9)  | 1    | 6.79                | 0.9093   | 0.3023   | -0.3773  |
|       | 2    | 5.79                | -0.0650  | -0.8598  | 0.2886   |
|       | 3    | 5.68                | 0.4112   | 0.4120   | 0.8799   |
| C(10) | 1    | 6.98                | 0.4002   | 0.0657   | -0.9032  |
|       | 2    | 5.56                | -0.8224  | -0.8405  | -0.2792  |
|       | 3    | 4.91                | 0.4045   | -0.5378  | 0.3260   |
| C(11) | 1    | 5.83                | 0.1855   | -0.1625  | -0.9374  |
|       | 2    | 4.83                | -0.4336  | -0.9794  | 0.1911   |
|       | 3    | 3.99                | 0.8818   | 0.1196   | 0.2911   |

The mean C-C distance and the average C-C-C angle in the phenyl ring are 1.401 Å and 120.0° respectively. The N-C and C-C distances in the pyrazolone ring lie between the corresponding single-bond and double-bond values. The N-CH<sub>3</sub> and C-CH<sub>3</sub> bond distances in the molecule are 1.494 ± 0.016 and 1.539

± 0.015 Å respectively and the length of the C-O bond in the carbonyl group is 1.284 ± 0.012 Å.

#### *The perchlorate ion*

The threefold axis of the tetrahedral perchlorate ion in the structure coincides with the threefold axis in the

Table 4. *Interatomic distances, valency angles and their standard deviations*

| Bond        | <i>l</i> | $\sigma(l)$ | Angle            | $\theta$ | $\sigma(\theta)$ |
|-------------|----------|-------------|------------------|----------|------------------|
| Ca—O(1)     | 2.304 Å  | 0.006 Å     | Ca—O(1)—C(1)     | 146.2°   | 0.6°             |
| O(1)—C(1)   | 1.284    | 0.012       | C(1)—C(2)—C(3)   | 107.0    | 0.9              |
| C(1)—C(2)   | 1.392    | 0.013       | C(2)—C(3)—C(4)   | 128.6    | 1.0              |
| C(2)—C(3)   | 1.357    | 0.016       | C(2)—C(3)—N(2)   | 111.4    | 0.9              |
| C(3)—C(4)   | 1.538    | 0.015       | C(4)—C(3)—N(2)   | 120.0    | 0.9              |
| C(3)—N(2)   | 1.368    | 0.011       | C(3)—N(2)—N(1)   | 106.2    | 0.8              |
| N(2)—C(5)   | 1.494    | 0.016       | C(3)—N(2)—C(5)   | 128.8    | 0.9              |
| N(2)—N(1)   | 1.407    | 0.012       | C(5)—N(2)—N(1)   | 119.7    | 0.8              |
| N(1)—C(1)   | 1.401    | 0.011       | N(2)—N(1)—C(6)   | 123.7    | 0.7              |
| N(1)—C(6)   | 1.424    | 0.010       | C(6)—N(1)—C(1)   | 127.4    | 0.8              |
| C(6)—C(7)   | 1.392    | 0.013       | N(2)—N(1)—C(1)   | 107.4    | 0.7              |
| C(7)—C(8)   | 1.420    | 0.014       | N(1)—C(1)—C(2)   | 107.9    | 0.8              |
| C(8)—C(9)   | 1.383    | 0.013       | N(1)—C(1)—O(1)   | 118.2    | 0.8              |
| C(9)—C(10)  | 1.393    | 0.016       | O(1)—C(1)—C(2)   | 133.5    | 0.9              |
| C(10)—C(11) | 1.413    | 0.015       | N(1)—C(6)—C(11)  | 120.2    | 0.8              |
| C(11)—C(6)  | 1.404    | 0.011       | N(1)—C(6)—C(7)   | 117.8    | 0.9              |
| Cl—O(2)     | 1.428    | 0.008       | C(11)—C(6)—C(7)  | 122.0    | 0.9              |
| Cl—O(3)     | 1.424    | 0.014       | C(6)—C(7)—C(8)   | 118.1    | 1.0              |
|             |          |             | C(7)—C(8)—C(9)   | 120.5    | 1.1              |
|             |          |             | C(8)—C(9)—C(10)  | 120.9    | 1.1              |
|             |          |             | C(9)—C(10)—C(11) | 119.8    | 1.1              |
|             |          |             | C(10)—C(11)—C(6) | 118.7    | 1.0              |
|             |          |             | O(2)—Cl—O(3)     | 110.6    | 0.8              |

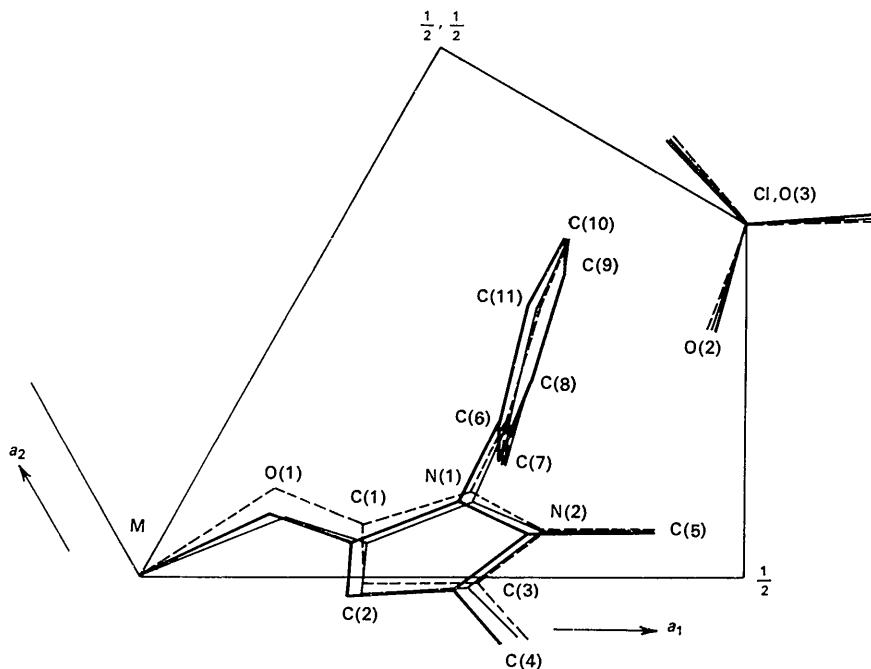


Fig. 3. Atomic positions in magnesium (heavy line), calcium (light line) and lead (broken line) hexa-antipyrine perchlorates as seen along the *c* axis.

space group, with one of the perchlorate oxygen atoms and the chlorine atom lying on it. The tetrahedron formed by the oxygen atoms is nearly regular, with only insignificant distortions. The non-equivalent Cl-O distances in the perchlorate group are  $1.428 \pm 0.008$  and  $1.424 \pm 0.014$  Å for Cl-O(2) and Cl-O(3) respectively. The corresponding distances corrected for thermal oscillations, using Busing & Levy's scheme (1964) assuming 'riding' motion, are respectively 1.479 and 1.472 Å.

The distances involved in the contacts between the perchlorate ion and the surrounding antipyrine groups and the intramolecular and intermolecular approaches involving the antipyrine groups correspond to the ex-

pected van der Waals interactions. Hence, none of them needs special comment.

### Metal-oxygen bonding and the molecular geometry of antipyrine in magnesium, calcium and lead hexa-antipyrine perchlorates

Atomic positions in the antipyrine group and the perchlorate ion in the structures, as viewed down the *c* and  $a_2^*$  axes, are shown in Figs. 3 and 4 respectively. The positional parameters of the respective atoms in the perchlorate ion in all the structures are comparable. The general orientation of the antipyrine molecule is

Table 5. Observed and calculated structure factors

| $\frac{h}{a}$     | $\frac{k}{a}$ | $\frac{l}{a}$ | $F_{\text{obs}}$ | $F_{\text{calc}}$ | $\frac{h}{a}$ | $\frac{k}{a}$ | $\frac{l}{a}$ | $F_{\text{obs}}$ | $F_{\text{calc}}$ | $\frac{h}{a}$ | $\frac{k}{a}$ | $\frac{l}{a}$ | $F_{\text{obs}}$ | $F_{\text{calc}}$ | $\frac{h}{a}$ | $\frac{k}{a}$ | $\frac{l}{a}$ | $F_{\text{obs}}$ | $F_{\text{calc}}$ |      |
|-------------------|---------------|---------------|------------------|-------------------|---------------|---------------|---------------|------------------|-------------------|---------------|---------------|---------------|------------------|-------------------|---------------|---------------|---------------|------------------|-------------------|------|
| $\frac{e}{a} = 0$ |               |               |                  |                   |               |               |               |                  |                   |               |               |               |                  |                   |               |               |               |                  |                   |      |
| 2 0 0             | 5607          | -1991         | 2 6 1            | 1157              | 1220          | -7 9          | 749           | 718              | 15 1              | 424           | 400           | 1 7           | 708              | 1056              | -5 9          | 696           | 745           | 7 6              | .765              | 148  |
| 3 0 0             | 5952          | 3292          | 2 7 9            | 862               | 691           | -7 10         | 749           | 621              | 15 2              | 387           | 356           | 1 8           | 1075             | 1010              | -5 10         | 538           | 591           | 7 7              | .470              | 159  |
| 4 0 0             | 1998          | 1889          | 2 7 9            | 612               | 585           | -7 12         | 729           | 655              | 14 1              | 207           | 187           | 1 10          | 626              | 507               | -5 11         | 666           | 796           | 8 2              | .850              | 564  |
| 5 0 0             | 809           | 947           | 2 7 9            | 574               | 574           | -7 14         | 559           | 492              | 14 2              | 205           | 209           | 1 11          | 759              | 774               | -5 12         | 552           | 604           | 8 3              | .417              | 197  |
| 6 0 0             | 1534          | 1511          | 3 5 7            | 5133              | 5349          | -9 12         | 559           | 559              | -1 2              | 586           | 586           | 1 12          | 594              | 466               | -5 13         | 447           | .596          | 8 4              | .417              | 227  |
| 7 0 0             | 606           | -940          | 3 5 7            | 405               | -8            | -9 10         | 1089          | 1091             | -1 1              | 306           | 1154          | 2 1           | 399              | 1058              | -4 6          | 1254          | 1202          | 8 5              | .409              | 105  |
| 8 0 0             | 469           | -455          | 3 4 6            | 846               | 949           | -9 12         | 793           | 740              | -1 4              | 255           | 2540          | 2 4 2         | 3251             | 3745              | -4 7          | 411           | .482          | 8 6              | .409              | 412  |
| 9 0 0             | 616           | 526           | 3 5 7            | 1244              | 1173          | -10 12        | 562           | 563              | -1 4              | 1355          | 1633          | 2 4 3         | 2540             | 2158              | -4 8          | 460           | .460          | 8 7              | .583              | 550  |
| 10 0              | 549           | -791          | 3 7 8            | 965               | 952           | -10 13        | 624           | 753              | -1 6              | 566           | 525           | 2 7           | 432              | 411               | -4 11         | 292           | .272          | 8 8              | .585              | 595  |
| 11 0              | 5581          | -3745         | 3 7 8            | 609               | 598           | -11 12        | 632           | 562              | -1 7              | 607           | -624          | 2 8           | 599              | -577              | -4 12         | 603           | .844          | 9 3              | .295              | 555  |
| 12 0              | 1561          | 1151          | 1 10 1           | 1023              | 1214          | -15 15        | 544           | 469              | -1 8              | 1187          | 1254          | 2 10          | 599              | 916               | -4 13         | 597           | .626          | 9 4              | .635              | 557  |
| 13 0              | 1104          | 1157          | 4 2 2            | 1831              | 1702          | -1 9          | 117           | 797              | -1 9              | 105           | 1207          | 2 11          | 727              | 1207              | -4 14         | 372           | .372          | 9 5              | .227              | 287  |
| 14 0              | 508           | -516          | 4 3 5            | 442               | -189          | 1 1           | 1668          | 1652             | -1 15             | 494           | 583           | 2 12          | 1477             | 1276              | -4 15         | 424           | .556          | 10 2             | .497              | 684  |
| 15 0              | 1554          | 1592          | 4 4 5            | 1255              | -1348         | 1 2 1         | 5158          | 5158             | -1 14             | 395           | 405           | 2 13          | 1477             | 1276              | -4 16         | 321           | .291          | 11 2             | .209              | 695  |
| 16 0              | 559           | 599           | 4 5 6            | 1187              | 1594          | 1 3 5         | 1256          | 2554             | -2 4              | 1488          | 1463          | 2 14          | 1472             | -1507             | -4 17         | 360           | .309          | 11 3             | .205              | 295  |
| 17 0              | 2103          | 2161          | 2 1 1            | 1151              | 1151          | 1 4 6         | 5064          | 5105             | -2 5              | 266           | 148           | 3 6           | 1505             | 1675              | -5 14         | 267           | .356          | 11 4             | .265              | 291  |
| 18 0              | 337           | 578           | 3 1 7            | 778               | 778           | 1 5 7         | 300           | 322              | -2 6              | 266           | 148           | 3 7           | 989              | -908              | -5 15         | 698           | .486          | 15 2             | .175              | 255  |
| 19 0              | 1153          | -1297         | 2 3 5            | 252               | 1914          | 1 6 7         | 1446          | 1523             | -2 7              | 1150          | 1259          | 3 8           | 505              | 626               | -5 16         | 865           | .865          | 15 3             | .121              | 181  |
| 20 0              | 1051          | 1069          | 5 3 5            | 1055              | 1050          | 1 7 7         | 1291          | 1258             | -2 8              | 767           | -679          | 3 9           | 523              | 622               | -5 17         | 401           | .395          | 1 2              | .186              | 109  |
| 21 0              | 1497          | 1555          | 5 4 6            | 1244              | 1135          | 1 9 7         | 605           | -547             | 2 10              | 912           | 1173          | 3 10          | 560              | -536              | -5 18         | 357           | .357          | 2 2              | .227              | 297  |
| 22 0              | 1017          | -894          | 5 6 6            | 598               | 622           | 2 1 11        | 152           | 146              | 2 11              | 640           | 640           | 4 3           | 422              | 512               | -5 19         | 1122          | .295          | 1 4              | .597              | 507  |
| 23 0              | 659           | 548           | 8 6 8            | 668               | 755           | 2 2 1         | 257           | 257              | 2 13              | 666           | 713           | 4 4           | 462              | -249              | -5 20         | 1664          | .1587         | 1 5              | .520              | 1299 |
| 24 0              | 249           | 478           | 2 4 7            | 612               | 487           | 2 3 5         | 721           | 708              | 2 14              | 614           | 619           | 4 5           | 734              | 781               | -5 21         | 360           | .451          | 1 10             | .599              | 667  |
| 25 0              | 1856          | 1879          | 6 3 5            | 1247              | 1247          | 2 4 6         | 829           | 739              | 2 15              | 190           | 157           | 4 6           | 449              | 605               | -5 22         | 542           | .441          | 1 11             | .101              | 1775 |
| 26 0              | 5178          | -2982         | 6 4 6            | 1716              | 1716          | 2 5 6         | 992           | 893              | 2 16              | 580           | 580           | 4 8           | 475              | 475               | -5 23         | 664           | .459          | 513              | .181              | 181  |
| 27 0              | 1255          | 1614          | 6 5 6            | 707               | 654           | 2 7 7         | 826           | -731             | 2 6 7             | 521           | -541          | 5 1           | 102              | 1767              | -5 24         | 1181          | .126          | 149              | .225              | 325  |
| 28 0              | 582           | -894          | 6 6 7            | 562               | 662           | 2 7 8         | 592           | -592             | 2 7 8             | 519           | -146          | 5 2           | 50               | 540               | -5 25         | 109           | .14           | 225              | .225              | 325  |
| 29 0              | 966           | 647           | 6 8 8            | 612               | 746           | 2 8 9         | 557           | -557             | 2 9 9             | 498           | 333           | 5 3           | 505              | 595               | -5 26         | 1115          | .1115         | 1115             | .1115             | 1115 |
| 30 0              | 1114          | 1465          | 7 1 7            | 1256              | 1071          | 2 10 10       | 519           | 519              | 2 10 10           | 486           | 504           | 5 4           | 507              | -452              | -5 27         | 1169          | .686          | 1169             | .686              | 1169 |
| 31 0              | 2161          | 2161          | 7 1 7            | 1256              | 1071          | 2 11 11       | 519           | 519              | 2 11 12           | 557           | 576           | 5 5           | 508              | 548               | -5 28         | 1172          | .545          | 1172             | .545              | 1172 |
| 32 0              | 570           | 526           | 7 3 5            | 1111              | 1090          | 2 12 12       | 508           | -508             | 2 12 13           | 566           | -566          | 5 6           | 510              | 522               | -5 29         | 1172          | .552          | 1172             | .552              | 1172 |
| 33 0              | 805           | 810           | 7 4 6            | 537               | 407           | 2 13 13       | 519           | 519              | 2 13 14           | 528           | 543           | 6 1           | 1183             | 1204              | -5 30         | 1254          | .422          | 10 10            | .599              | 360  |
| 34 0              | 1074          | 1132          | 8 1 6            | 1293              | 1165          | 2 14 14       | 524           | 444              | 2 14 15           | 424           | 456           | 6 2           | 1063             | -149              | -5 31         | 1096          | .242          | 10 11            | .722              | 708  |
| 35 0              | 1559          | -285          | 8 2 6            | 639               | 610           | 2 15 15       | 562           | 562              | 2 15 16           | 588           | 523           | 6 3           | 515              | 505               | -5 32         | 162           | .267          | 12 12            | .592              | 568  |
| 36 0              | 666           | 544           | 8 4 7            | 741               | 741           | 2 16 16       | 529           | 529              | 2 16 17           | 497           | 519           | 6 4           | 567              | 557               | -5 33         | 1256          | .567          | 12 13            | .476              | 576  |
| 37 0              | 534           | 454           | 9 1 7            | 525               | -500          | 2 17 17       | 526           | 526              | 2 17 18           | 547           | 547           | 6 5           | 570              | 570               | -5 34         | 1257          | .567          | 12 14            | .476              | 576  |
| 38 0              | 950           | -781          | 9 2 6            | 616               | 591           | 3 10          | 375           | 414              | 3 10 11           | 448           | 448           | 6 6           | 505              | 505               | -5 35         | 1115          | .567          | 11 16            | .567              | 1115 |
| 39 0              | 4024          | 4181          | 9 3 7            | 466               | 391           | 4 1 12        | 1346          | -1419            | 4 1 13            | 284           | 310           | 6 7           | 506              | 534               | -5 36         | 1149          | .490          | 11 17            | .479              | 479  |
| 40 0              | 975           | 899           | 9 2 9            | 1092              | 1166          | 4 2 10        | 830           | 830              | 4 2 11            | 261           | 261           | 7 1           | 523              | 523               | -5 37         | 1202          | .207          | 11 18            | .207              | 102  |
| 41 0              | 1405          | 1496          | 9 5 7            | 907               | 866           | 4 3 12        | 2406          | 2410             | 4 3 13            | 616           | 1270          | 7 2           | 516              | 516               | -5 38         | 1204          | .590          | 11 19            | .590              | 374  |
| 42 0              | 799           | 815           | 9 6 8            | 1241              | 1241          | 5 1 13        | 526           | 526              | 5 1 14            | 565           | 1239          | 7 3           | 694              | 620               | -5 39         | 1166          | .265          | 11 20            | .265              | 1166 |
| 43 0              | 559           | 249           | 10 2 9           | 795               | 767           | 5 2 14        | 529           | 529              | 5 2 15            | 566           | 260           | 7 4           | 503              | 300               | -5 40         | 1244          | .224          | 11 21            | .224              | 224  |
| 44 0              | 1109          | 1163          | 10 3 10          | 602               | 574           | 5 3 15        | 1346          | 1305             | 5 3 16            | 421           | 422           | 7 5           | 441              | 441               | -5 41         | 1041          | .223          | 11 22            | .223              | 223  |
| 45 0              | 679           | -472          | 10 4 11          | 1801              | 1606          | 5 4 16        | 541           | 434              | 5 4 17            | 527           | 527           | 7 6           | 404              | 422               | -5 42         | 1041          | .222          | 11 23            | .222              | 222  |
| 46 0              | 354           | 285           | 2 4 5            | 400               | -300          | 5 5 17        | 528           | 542              | 5 5 18            | 1644          | 1554          | 10 1          | 516              | -293              | -5 43         | 999           | .746          | 8 8              | .578              | 191  |
| 47 0              | 822           | 700           | 2 4 6            | 390               | -390          | 5 6 18        | 527           | 527              | 5 6 19            | 530           | 530           | 10 2          | 516              | 529               | -5 44         | 1265          | .409          | 11 24            | .409              | 409  |
| 48 0              | 604           | 648           | 2 5 7            | 328               | 255           | 5 7 11        | 201           | 204              | 5 7 12            | 572           | 572           | 10 3          | 515              | 525               | -5 45         | 1035          | .1036         | 11 25            | .1036             | 1036 |
| 49 0              | 717           | 753           | 2 6 8            | 2478              | 2353          | 6 1 12        | 190           | -219             | 6 1 13            | 530           | 530           | 10 4          | 2076             | 2057              | -5 46         | 1951          | .1959         | 11 26            | .1959             | 1275 |
| 50 0              | 559           | 578           | 2 8 9            | 1498              | 1424          | 6 2 13        | 421           | 254              | 6 2 14            | 521           | 521           | 11 1          | 564              | 572               | -5 47         | 2151          | .2142         | 11 27            | .2142             | 1277 |
| 51 0              | 515           | -280          | 6 3 10           | 540               | 505           | 6 3 11        | 519           | -587             | 6 3 12            | 410           | 459           | 11 2          | 523              | 521               | -5 48         | 1121          | .455          | 11 28            | .455              | 65   |
| 52 0              | 359           | 261           | 6 4 11           | 524               | 524           | 6 4 12        | 445           | -1467            | 6 4 13            | 533           | 590           | 11 3          | 576              | 576               | -5 49         | 421           | .421          | 11 29            | .421              | 355  |
| 53 0              | 1559          | 261           | 6 5 12           | 1055              | 1083          | 6 5 13        | 405           | 516              | 6 5 14            | 277           | 356           | 11 4          | 523              | 523               | -5 50         | 297           | .297          | 11 30            | .297              | 359  |
| 54 0              | 815           | 792           | 6 6 13           | 624               | 594           | 6 6 14        | 405           | 516              | 6 6 15            | 448           | 409           | 11 5          | 503              | 503               | -5 51         | 1041          | .448          | 11 31            | .448              | 448  |
| 55 0              | 359           | 283           | 6 7 14           | 1826              | 1799          | 6 7 15        | 325           | 325              | 6 7 16            | 290           | 410           | 11 6          | 516              | 486               | -5 52         | 1254          | .486          | 11 32            | .486              | 486  |
| 56 0              | 102           | 109           | 6 8 15           | 493               | 495           | 6 8 16        | 828           | 699              | 6 8 17            | 517           | 509           | 11 7          | 516              | 516               | -5 53         | 1252          | .486          | 11 33            | .486              | 486  |
| 57 0              | 109</         |               |                  |                   |               |               |               |                  |                   |               |               |               |                  |                   |               |               |               |                  |                   |      |

Table 5 (cont.)

| <i>b</i> | <i>k</i> | $\tau_{\text{obs}}$ | $\tau_{\text{calo}}$ | <i>b</i> | <i>l</i> | $\tau_{\text{obs}}$ | $\tau_{\text{calo}}$ |
|----------|----------|---------------------|----------------------|----------|----------|---------------------|----------------------|----------|----------|---------------------|----------------------|----------|----------|---------------------|----------------------|----------|----------|---------------------|----------------------|
| 1        | 9        | 170                 | 1186                 | -5       | 4        | 504                 | 640                  | 5        | 1        | 408                 | 150                  | -8       | 15       | 275                 | 350                  | 4        | 5        | 2095                | 2064                 |
| 1        | 10       | 157                 | 421                  | -5       | 5        | 370                 | 391                  | 2        | 7        | 807                 | 107                  | -9       | 12       | 595                 | 409                  | 5        | 5        | 406                 | 452                  |
| 1        | 14       | 209                 | 224                  | -5       | 7        | 233                 | -355                 | 3        | 4        | 116                 | 116                  | -5       | 8        | 789                 | 875                  | 4        | 6        | 406                 | 452                  |
| 2        | 2        | 205                 | -324                 | -5       | 8        | 725                 | 794                  | 5        | 5        | 1459                | 1504                 | -10      | 11       | 637                 | 565                  | 10       | 10       | 556                 | 475                  |
| 2        | 5        | 445                 | -455                 | -5       | 11       | 1155                | 1058                 | 5        | 6        | 116                 | 511                  | -11      | 11       | 579                 | 424                  | 5        | 11       | 582                 | 551                  |
| 2        | 6        | 316                 | 396                  | -5       | 12       | 107                 | 189                  | 5        | 9        | 264                 | 524                  | -11      | 14       | 524                 | 274                  | 12       | 12       | 279                 | 418                  |
| 2        | 9        | 272                 | 297                  | -5       | 13       | 1265                | 1268                 | 5        | 11       | 59                  | 527                  | -11      | 14       | 154                 | 215                  | 14       | 14       | 262                 | 341                  |
| 2        | 13       | 251                 | 277                  | -5       | 14       | 850                 | 875                  | 6        | 1        | 640                 | 640                  | -11      | 14       | 247                 | 247                  | 15       | 15       | 1453                | 1453                 |
| 5        | 1        | 1661                | 1581                 | -4       | 7        | 1110                | 1175                 | 6        | 3        | 979                 | 1056                 | -4       | 6        | 589                 | 459                  | 4        | 4        | 604                 | 592                  |
| 5        | 2        | 1486                | 1575                 | -4       | 8        | 654                 | -708                 | 6        | 4        | 916                 | 814                  | -4       | 7        | 557                 | 584                  | 5        | 5        | 916                 | 976                  |
| 5        | 5        | 1608                | -1518                | -4       | 11       | 509                 | 289                  | 6        | 5        | 575                 | 404                  | 1        | 2        | 1044                | 1115                 | 4        | 6        | 673                 | 852                  |
| 5        | 6        | 1166                | 1010                 | -5       | 6        | 743                 | 674                  | 6        | 6        | 558                 | 554                  | 1        | 4        | 924                 | 994                  | 4        | 10       | 694                 | 751                  |
| 5        | 7        | 1276                | 1017                 | -5       | 7        | 1057                | 1056                 | 6        | 9        | 106                 | 261                  | 1        | 4        | 459                 | 194                  | 10       | 10       | 203                 | 252                  |
| 5        | 6        | 664                 | -595                 | -5       | 8        | 975                 | 853                  | 7        | 7        | 640                 | 626                  | 6        | 7        | 275                 | 59                   | 5        | 7        | 454                 | 425                  |
| 5        | 7        | 750                 | 558                  | -5       | 12       | 408                 | 446                  | 7        | 2        | 793                 | 796                  | 1        | 11       | 256                 | 149                  | 5        | 9        | 958                 | 1043                 |
| 5        | 8        | 800                 | 800                  | -5       | 16       | 172                 | -25                  | 7        | 3        | 452                 | 404                  | 2        | 1        | 1292                | 1158                 | 5        | 12       | 458                 | 566                  |
| 4        | 1        | 597                 | 296                  | -6       | 8        | 991                 | 1002                 | 7        | 7        | 272                 | 583                  | 2        | 2        | 675                 | 671                  | 5        | 13       | 310                 | 360                  |
| 4        | 2        | 706                 | 714                  | -6       | 10       | 1060                | 1055                 | 8        | 1        | 452                 | 504                  | 2        | 5        | 275                 | 304                  | 6        | 6        | 625                 | 985                  |
| 4        | 3        | 1142                | 1168                 | -5       | 12       | 599                 | 520                  | 8        | 2        | 641                 | 567                  | 2        | 4        | 605                 | 527                  | 6        | 8        | 699                 | 676                  |
| 4        | 4        | 586                 | 592                  | -5       | 13       | 581                 | 580                  | 8        | 4        | 299                 | 513                  | 2        | 6        | 419                 | 524                  | 7        | 8        | 514                 | 347                  |
| 4        | 5        | 429                 | -592                 | -6       | 14       | 452                 | 452                  | 8        | 8        | 195                 | 287                  | 2        | 7        | 867                 | 861                  | 7        | 9        | 542                 | 445                  |
| 4        | 6        | 1104                | 900                  | -7       | 10       | 693                 | 666                  | 9        | 2        | 506                 | -278                 | 2        | 10       | 251                 | 196                  | 7        | 10       | 586                 | 517                  |
| 4        | 7        | 697                 | 555                  | -7       | 11       | 221                 | 458                  | 9        | 5        | 264                 | 281                  | 2        | 11       | 228                 | 121                  | 9        | 10       | 431                 | 527                  |
| 4        | 8        | 597                 | 414                  | -7       | 13       | 112                 | 155                  | 9        | 4        | 272                 | 164                  | 2        | 12       | 146                 | 182                  | 8        | 12       | 422                 | 506                  |
| 4        | 9        | 408                 | 563                  | -7       | 14       | 408                 | 480                  | 9        | 6        | 285                 | 155                  | 4        | 1        | 977                 | 1012                 | 10       | 1        | 448                 | 444                  |
| 4        | 12       | 288                 | 562                  | -7       | 15       | 506                 | 550                  | 10       | 1        | 125                 | 521                  | 1        | 2        | 1045                | 1121                 | 10       | 5        | 725                 | 466                  |
| 5        | 1        | 1205                | 1573                 | -7       | 16       | 180                 | 118                  | 10       | 4        | 212                 | 157                  | 1        | 4        | 791                 | 860                  | 9        | 15       | 259                 | 227                  |
| 5        | 3        | 715                 | 618                  | -6       | 9        | 625                 | 564                  | 10       | 5        | 484                 | 504                  | 5        | 5        | 1505                | 1496                 | 9        | 2        | 615                 | 746                  |
| 5        | 6        | 755                 | 624                  | -6       | 10       | 274                 | -175                 | 11       | 2        | 236                 | 251                  | 4        | 6        | 456                 | 29                   | 5        | 9        | 405                 | 556                  |
| 5        | 7        | 541                 | 589                  | -6       | 11       | 171                 | 178                  | 12       | 3        | 210                 | 119                  | 5        | 8        | 120                 | 400                  | 10       | 10       | 199                 | 254                  |
| 6        | 1        | 740                 | 604                  | -5       | 15       | 311                 | 420                  | 15       | 1        | 201                 | 169                  | 10       | 17       | 447                 | 447                  | 11       | 12       | 209                 | 515                  |
| 6        | 2        | 862                 | 928                  | -9       | 10       | 505                 | -261                 | -1       | 7        | 741                 | 693                  | 4        | 1        | 104                 | 142                  | -11      | 15       | 260                 | 563                  |
| 6        | 5        | 490                 | -422                 | -9       | 11       | 612                 | 565                  | -5       | 9        | 459                 | 156                  | 2        | 2        | 260                 | 455                  | 12       | 15       | 266                 | 309                  |
| 6        | 4        | 272                 | -253                 | -9       | 15       | 249                 | 251                  | -1       | 6        | 285                 | 155                  | 4        | 1        | 977                 | 1012                 | 10       | 5        | 714                 | 784                  |
| 6        | 5        | 1058                | 788                  | -10      | 11       | 155                 | -288                 | -1       | 7        | 407                 | 129                  | 4        | 4        | 452                 | 426                  | 10       | 5        | 614                 | 585                  |
| 7        | 5        | 596                 | 596                  | -10      | 12       | 217                 | 396                  | -1       | 8        | 811                 | 812                  | 4        | 5        | 422                 | 176                  | 9        | 2        | 600                 | 576                  |
| 7        | 5        | 615                 | 731                  | -10      | 16       | 216                 | 205                  | -1       | 9        | 452                 | 411                  | 4        | 5        | 215                 | 172                  | 10       | 9        | 414                 | 398                  |
| 7        | 6        | 512                 | 524                  | -11      | 14       | 312                 | 400                  | -1       | 10       | 596                 | 521                  | 5        | 3        | 154                 | -32                  | -11      | 2        | 707                 | 782                  |
| 7        | 7        | 261                 | 540                  | -11      | 15       | 205                 | 505                  | -1       | 11       | 268                 | -176                 | 5        | 4        | 279                 | 260                  | 0        | 5        | 154                 | 255                  |
| 8        | 1        | 752                 | -768                 | -12      | 14       | 281                 | 291                  | -1       | 14       | 201                 | 219                  | 5        | 4        | 578                 | 572                  | 0        | 4        | 1425                | 1425                 |
| 8        | 5        | 624                 | 912                  | -11      | 15       | 242                 | 266                  | -2       | 5        | 1126                | -143                 | 5        | 5        | 277                 | -235                 | 0        | 5        | 578                 | 578                  |
| 8        | 4        | 457                 | 457                  | -12      | 16       | 87                  | 254                  | -2       | 6        | 472                 | 440                  | 5        | 9        | 157                 | 155                  | 0        | 6        | 1863                | 2020                 |
| 8        | 5        | 365                 | -523                 | -12      | 15       | 174                 | 192                  | -2       | 6        | 394                 | -199                 | 1        | 1        | 501                 | 501                  | 0        | 7        | 418                 | 418                  |
| 8        | 7        | 516                 | 404                  | -6       | 14       | 214                 | 214                  | -2       | 7        | 569                 | 528                  | 6        | 2        | 817                 | 295                  | 0        | 7        | 497                 | 497                  |
| 9        | 1        | 410                 | -349                 | -6       | 15       | 344                 | 406                  | -2       | 8        | 815                 | 729                  | 6        | 4        | 510                 | 162                  | 0        | 10       | 612                 | 622                  |
| 9        | 2        | 766                 | 665                  | 1        | 1        | 1501                | 1366                 | -2       | 10       | 670                 | 576                  | 6        | 5        | 455                 | 486                  | 0        | 11       | 155                 | -228                 |
| 9        | 3        | 557                 | 652                  | 1        | 2        | 1659                | 1718                 | -2       | 11       | 457                 | 265                  | 6        | 8        | 518                 | 341                  | 1        | 2        | 1805                | -245                 |
| 9        | 4        | 884                 | 554                  | 1        | 4        | 552                 | 402                  | -2       | 12       | 170                 | 184                  | 6        | 8        | 518                 | 341                  | 1        | 4        | 1042                | 1116                 |
| 9        | 7        | 196                 | 518                  | 1        | 5        | 551                 | 496                  | -4       | 4        | 214                 | 214                  | 7        | 1        | 706                 | 624                  | 1        | 5        | 1477                | 1451                 |
| 10       | 1        | 592                 | 516                  | 1        | 7        | 567                 | 587                  | -5       | 5        | 469                 | 592                  | 7        | 6        | 245                 | 162                  | 1        | 4        | 2004                | 1584                 |
| 10       | 5        | 746                 | 809                  | 1        | 8        | 815                 | 651                  | -6       | 6        | 1418                | 1515                 | 7        | 7        | 292                 | 236                  | 2        | 1        | 2515                | 2814                 |
| 10       | 4        | 247                 | 277                  | 1        | 9        | 719                 | 859                  | -7       | 7        | 531                 | -159                 | 8        | 1        | 551                 | 548                  | 1        | 8        | 569                 | 425                  |
| 10       | 5        | 216                 | 236                  | 1        | 10       | 259                 | 429                  | -8       | 8        | 535                 | -106                 | 8        | 4        | 441                 | 446                  | 1        | 9        | 718                 | 651                  |
| 11       | 1        | 505                 | 265                  | 1        | 11       | 214                 | 154                  | -9       | 9        | 764                 | 764                  | 9        | 2        | 285                 | 159                  | 2        | 1        | 1595                | 1520                 |
| 11       | 5        | 547                 | 462                  | 2        | 1        | 542                 | 580                  | -15      | 15       | 151                 | 144                  | 9        | 2        | 285                 | 179                  | 2        | 5        | 1227                | 1121                 |
| 11       | 4        | 181                 | 552                  | 2        | 2        | 778                 | 772                  | -15      | 5        | 1210                | 1110                 | 9        | 4        | 178                 | 265                  | 1        | 6        | 619                 | -673                 |
| 12       | 2        | 222                 | 214                  | 2        | 5        | 344                 | 406                  | -16      | 6        | 862                 | 848                  | 11       | 1        | 267                 | 261                  | 2        | 4        | 2216                | 2211                 |
| 12       | 4        | 211                 | 244                  | 2        | 4        | 275                 | -105                 | -7       | 504      | 561                 | 12                   | 1        | 194      | 244                 | 2                    | 5        | 454      | -564                |                      |
| 12       | 5        | 175                 | 160                  | 2        | 5        | 151                 | 145                  | -16      | 4        | 446                 | 466                  | 12       | 2        | 222                 | 222                  | 3        | 6        | 1271                | 1241                 |
| 12       | 5        | 716                 | -688                 | 2        | 7        | 578                 | 580                  | -14      | 9        | 1100                | 1099                 | -1       | 2        | 488                 | 612                  | 2        | 7        | 878                 | 895                  |
| -1       | 4        | 2520                | 2105                 | 2        | 8        | 1024                | 951                  | -14      | 11       | 171                 | 150                  | -1       | 3        | 812                 | 724                  | 2        | 7        | 977                 | 977                  |
| -1       | 5        | 458                 | 475                  | 2        | 11       | 559                 | 416                  | -7       | 7        | 158                 | 1458                 | -1       | 5        | 941                 | -614                 | 2        | 9        | 749                 | 749                  |
| -1       | 6        | 563                 | -253                 | 5        | 2        | 623                 | -565                 | -5       | 9        | 157                 | 741                  | -1       | 6        | 294                 | 298                  | 5        | 1        | 5170                | -418                 |
| -1       | 7        | 419                 | 517                  | 5        | 5        | 481                 | 561                  | -9       | 10       | 477                 | 477                  | -1       | 7        | 751                 | 614                  | 5        | 2        | 977                 | 674                  |
| -1       | 8        | 558                 | 267                  | 5        | 5        | 566                 | -484                 | -17      | 11       | 496                 | 611                  | -1       | 9        | 485                 | 546                  | 4        | 5        | 1814                | 1916                 |
| -1       | 10       | 597                 | 565                  | 5        | 6        | 929                 | 884                  | -15      | 12       | 419                 | 419                  | -1       | 11       | 484                 | 748                  | 5        | 4        | 2570                | 2797                 |
| -1       | 15       | 194                 | 258                  | 5        | 7        | 404                 | 199                  | -15      | 12       | 429                 | 542                  | -1       | 11       | 247                 | 224                  | 5        | 6        | 609                 | 770                  |
| -2       | 5        | 1055                | 1101                 | 5        | 8        | 459                 | -155                 | -6       | 9        | 571                 | 662                  | -1       | 12       | 259                 | 236                  | 5        | 6        | 1271                | 1241                 |
| -2       | 5        | 1037                | 968                  | 5        | 10       | 511                 | 400                  | -6       | 12       | 713                 | 740                  | -1       | 11       | 110                 | 119                  | 5        | 7        | 191                 | -453                 |
| -2       | 6        | 862                 | 934                  | 5        | 12       | 151                 | 161                  | -16      | 13       | 150                 | 451                  | -2       | 5        | 1699                | 164                  | 5        | 8        | 173                 | -560                 |
| -2       | 7        | 559                 | -64                  | 4        | 13       | 1211                | 1205                 | -7       | 8        | 561                 | 447                  | -2       | 4        | 255                 | -156                 | 5        | 9        | 597                 | -375                 |
| -2       | 8        | 1072                | 1172                 | 4        | 14       | 427                 | 116                  | -9       | 10       | 561                 | 447                  | -2       | 5        | 536                 | -509                 | 5        | 10       | 467                 | -537                 |
| -2       | 9        | 514                 | 464                  | 4        | 5        | 455                 | -169                 | -7       | 10       | 621                 | 721                  | -2       | 4        | 411                 | 441                  | 4        | 7        | 799                 | 788                  |
| -2       | 11       | 586                 | 419                  | 4        | 6        | 619                 | 471                  | -7       | 11       | 596                 | 602                  | -2       | 10       | 454                 | 540                  | 4        | 8        | 749                 | 765                  |
| -2       | 14       | 222                 | 525                  | 4        | 5        | 516                 | 109                  | -7       | 14       | 246</               |                      |          |          |                     |                      |          |          |                     |                      |

Table 6. Equations of the mean planes of the phenyl and pyrazolone rings

The displacements ( $\text{\AA}$ ) of the atoms from, and their standard deviations ( $\text{\AA}$ ) normal to, the planes are also indicated.  
Phenyl ring:

$$0.9488 X' - 0.3055 Y' - 0.0798 Z' - 2.8470 = 0.$$

|       | $\Delta$ | $\sigma_{\perp}$ |
|-------|----------|------------------|
| C(6)  | 0.001    | 0.010            |
| C(7)  | -0.002   | 0.011            |
| C(8)  | -0.002   | 0.013            |
| C(9)  | 0.007    | 0.012            |
| C(10) | -0.008   | 0.011            |
| C(11) | 0.004    | 0.011            |

$$\chi^2 = 1.08; \quad v = 3, \quad P < 80\% \text{ and} > 70\%$$

Pyrazolone ring:

$$-0.2458 X' + 0.8623 Y' - 0.4428 Z' + 3.6334 = 0.$$

|      | $\Delta$ | $\sigma_{\perp}$ |
|------|----------|------------------|
| N(1) | -0.020   | .007             |
| N(2) | 0.021    | .008             |
| C(1) | 0.012    | .009             |
| C(2) | 0.001    | .010             |
| C(3) | -0.014   | .011             |

$$\chi^2 = 18.37; \quad v = 2; \quad P < 0.1\%$$

nearly the same in the structures although individual atomic positions show appreciable deviations. As is to be expected, deviations are maximum for the coordinating oxygen atom and are quite considerable for the other atoms in the pyrazolone group. The differences in the atomic positions of the phenyl carbon atoms in the three compounds, though not as large as those in the case of the pyrazolone ring, are not insignificant.

#### Coordination of the metal ion

In these compounds, the  $\bar{3}$  equivalent antipyrine molecules are coordinated to the central metal ion through the lone oxygen atom in the carbonyl group. As mentioned earlier, the octahedral coordination of these oxygen atoms about the metal ion is hence determined essentially by the symmetry of the space group. The  $\bar{3}$  axis of this octahedron coincides with that of the space group and the only permissible departure from a perfect symmetrical arrangement is an elongation or a contraction in the direction of the  $z$  axis.

The principal distances and angles involved in the arrangement of oxygen atoms around the metal ion

are given in Table 7. For this arrangement to be a regular octahedron, all the OMO angles ( $M = \text{Mg}^{2+}$ ,  $\text{Ca}^{2+}$  or  $\text{Pb}^{2+}$ ) should be exactly  $90^\circ$  and all the O-O distances should be equal, and any departure from these conditions is a measure of the distortion of the regular figure. Further, for a regular octahedron,  $c/a = \sqrt{2}/2$  where  $c$  is the height of the octahedron along the  $z$  axis and  $a$  is the centroid-to-apex distance of the triangular face perpendicular to it. This ratio is greater or less than the ideal value of  $\sqrt{2}/2$  according as the polyhedron is elongated or contracted along the  $\bar{3}$  axis. The  $c/a$  ratios in the three cases are also listed in Table 7. An examination of this Table reveals that the octahedron is regular within experimental error in the magnesium compound but becomes elongated in the  $z$  direction as the size of the metal ion increases. In the absence of any chemical reason warranting such an elongation, it may be considered as due to packing requirements resulting from the increase in the size of the central metal ion, and it takes place in the  $z$  direction probably because of the less dense packing of molecules along the  $c$  axis on either side of the octahedron than in other directions.

The coordination polyhedron of oxygen atoms around divalent magnesium is almost invariably octahedral whereas the number of oxygen atoms that coordinate divalent calcium varies from 6 to 12. However, in both cases, the coordination is determined essentially by geometrical and packing considerations and the binding forces are predominantly electrostatic in character. The plumbous ion, on the other hand, is known to exhibit a wide variety of coordination numbers ranging from 3 to 12 and it is generally rec-

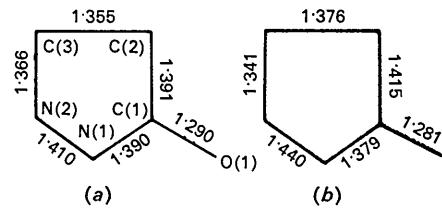


Fig. 6. (a) Observed bond lengths ( $\text{\AA}$ ) in the pyrazolone ring.  
(b) Bond lengths ( $\text{\AA}$ ) calculated from the estimated relative contributions of the canonical structures.

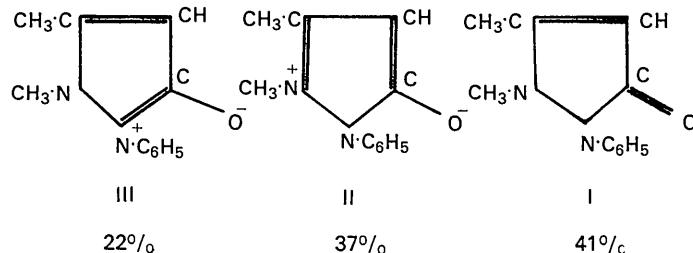


Fig. 5. Canonical structures of antipyrine and their relative contributions.

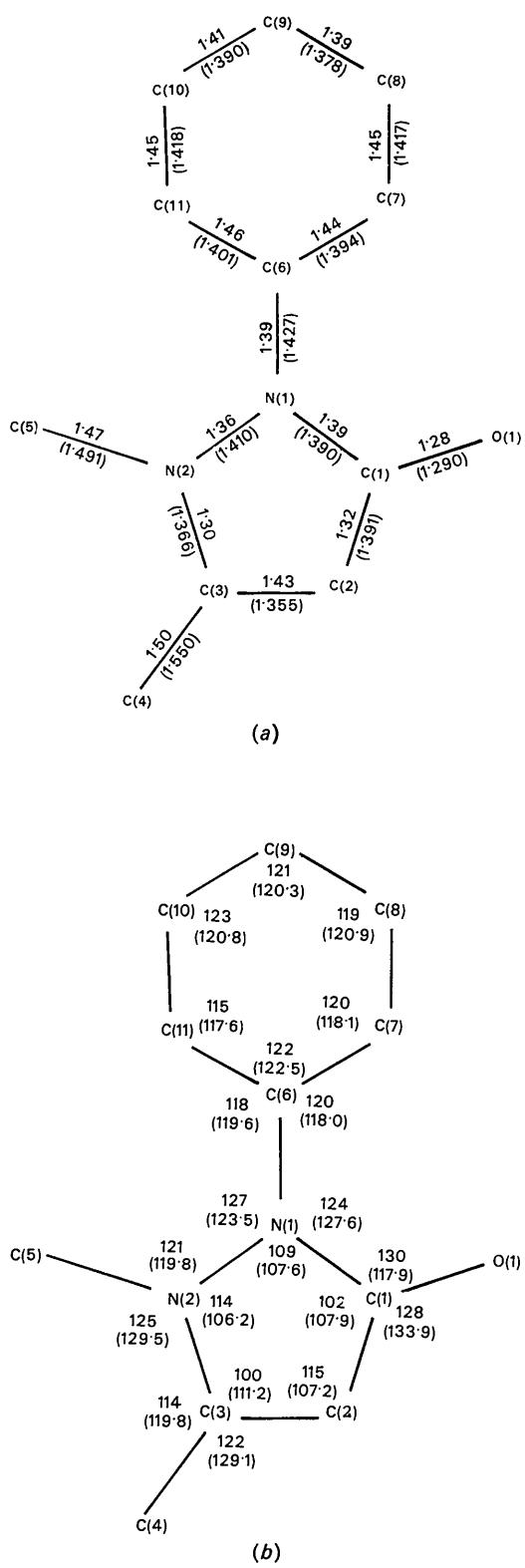


Fig. 7. (a) Bond lengths ( $\text{\AA}$ ) and (b) bond angles ( $^\circ$ ) in 4-bromoantipyrine. The corresponding values obtained from the present analysis are given in parentheses.

Table 7. Principal lengths ( $\text{\AA}$ ) and angles ( $^\circ$ ) in the coordination polyhedron around the metal ion in  $M(C_{11}H_{12}ON_2)_6(ClO_4)_2$

**M = Mg, Ca, or Pb**

|   | Mg    | Ca    | Pb    |
|---|-------|-------|-------|
| M—O(1 <sup>1</sup> )                      | 2.059 | 2.304 | 2.446 |
| O(1 <sup>ii</sup> )—O(1 <sup>iii</sup> )  | 2.920 | 3.215 | 3.319 |
| O(1 <sup>1</sup> )—O(1 <sup>ii</sup> )    | 2.905 | 3.310 | 3.595 |
| O(1 <sup>1</sup> )—M—O(1 <sup>iii</sup> ) | 90.3  | 88.1  | 85.4  |
| O(1 <sup>1</sup> )—M—O(1 <sup>ii</sup> )  | 89.7  | 91.9  | 94.6  |
| <i>c/a</i>                                | 1.403 | 1.480 | 1.588 |

- i       $x, y, z$
- ii      $y, y-x, \bar{z}$
- iii     $x-y, x, \bar{z}$

ognized that the Pb<sup>II</sup>-O bond has an appreciable covalent nature (Vijayan & Viswamitra, 1966).

The approximate amount of ionic character of a bond between atoms A and B with electronegativity coefficients  $x_A$  and  $x_B$  can be estimated by use of the equation

Amount of ionic character =

$$0.16(x_A - x_B) + 0.035(x_A - x_B)^2$$

(Hanny & Smyth, 1946). The ionic character of the  $Mg^{II}-O$ ,  $Ca^{II}-O$  and  $Pb^{II}-O$  bonds, derived from the above equation and the known electronegativity coefficients of the relevant atoms (Pauling, 1960; Gordy & Thomas, 1956), are 56.3%, 61.8% and 43% respectively. These values suggest that the first two bonds are predominantly ionic whereas the third is predominantly covalent.

### *The antipyrine group*

The orientation of the antipyrine group and its dimensions are similar in all three compounds. However, the estimated bond lengths and valency angles in the lead compound are much less accurate than those in the other two and hence the dimensions of the antipyrine group used in the following discussion are, unless otherwise stated, the average values of those obtained from the analysis of the magnesium and the calcium compounds.

The antipyrine molecule can be considered as a resonance hybrid, the three important canonical structures of which are given in Fig. 5 (Knorr, 1887; Michaelis, 1902; Kitamura, 1940; Valyashko & Bliznyukov, 1941; Brown, Hukins, Le Fevre, Northcott & Wilson, 1949). The estimates of their relative contributions which explain most satisfactorily the observed bond lengths are also indicated. The relevant bond lengths calculated on the basis of these contributions and the corresponding observed values are shown in Fig. 6. The C-C distances were calculated by means of the semi-empirical formula

$$D_n = D_1 - (D_1 - D_2) \frac{1.84(n-1)}{0.84n + 0.16}$$

(Pauling, 1960) in which  $D_n$  is the value of the interatomic bond distance of intermediate type,  $D_1$  that for a single bond,  $D_2$  that for a double bond and  $n$  the bond number. The C–N and C–O distances were obtained from the curves given by Donohue, Lavine & Rollett (1956) and Curl (1959) respectively. Calculations based on the C–N and the C–O distances given in Table 6 of Marsh, Bierstedt & Eichhorn (1962) resulted in slightly different relative contributions of the canonical structures (37% of I, 39% of II and 24% of III). However, the values of Donohue *et al.* and Curl were thought to be more reliable as they have used a larger number of observations in drawing up the bond length–bond order curves. The N(1)–N(2) bond is single in all the three canonical structures and the relatively high discrepancy between its observed and calculated lengths may be due to the neglect of other less important resonance structures.

The bond length–bond order curves used in the above calculations have no firm theoretical basis and the estimated contributions of the canonical forms cannot be considered to be accurate. However, they are of interest as they give a semi-quantitative idea of the relative importance of the different structures proposed. The preponderance of the phenylhydrazine structure by Knorr (I) is expected since it does not involve charge separation. The phenol betaine structure of Michaelis (II) is second in importance whereas the contribution of the third structure proposed by Kitamura (III) is substantial. However, it might be mentioned that the association of the metal ion at the carbonyl oxygen favours the development of the ionic forms and hence the contribution of Knorr's structure can be expected to be larger in free antipyrine than in its complexes.

The pyrazolone ring and the phenyl ring are tilted with respect to each other by 62·3, 62·5 and 68·6° in the magnesium, the calcium and the lead compounds respectively. The respective distances of the N(1)–C(6) bonds which connect the phenyl ring to the pyrazolone ring, *viz.*  $1\cdot430 \pm 0\cdot011$ ,  $1\cdot424 \pm 0\cdot010$  and  $1\cdot414 \pm 0\cdot024$  Å, are not significantly different from the expected N–C distance of  $1\cdot440$  Å for  $sp^2$  hybridized carbon (Dewar & Schmeising, 1959). The single-bond character of the N–C link permits the orientation of the phenyl group to be determined entirely by steric considerations. The actual tilts are probably determined, as was shown in the communication on the structure of the magnesium compound (Vijayan & Viswamitra, 1967), by intramolecular and intermolecular steric effects.

The only X-ray investigation on the derivatives of antipyrine so far reported is the determination of the structure of 4-bromoantipyrine by two-dimensional methods (Romain, 1957) and it is of interest to compare the molecular geometry of antipyrine deduced from this analysis with that obtained from our studies. The molecular dimensions of 4-bromoantipyrine are shown in Fig. 7. The corresponding lengths and angles obtained from the present analysis are also indicated for comparison. Even a cursory look at these values

reveals that the dimensions obtained by Romain are substantially different from those obtained by us. It is rather improbable that the substitution of bromine in the 4-position on the one hand and the association of the metal ion on the other are responsible for such drastic changes in bond lengths in the pyrazolone group. It is felt that these differences are probably due to the fact that the structure of 4-bromoantipyrine, determined from two-dimensional data, merits further refinement.

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